Electric field perpendicular to magnetic field

It isn't. Consider, for example, an infinite horizontal sheet of positive charge with an infinite current-carrying wire, parallel to the sheet, suspended above it. The electric field from the charged sheet points upward everywhere above it. The magnetic field curls around the wire. This means that, in the plane that contains the wire and is parallel to the charged sheet, the magnetic field is parallel (or antiparallel) to the electric field in this case. Next: Exercises Up: Multi-Dimensional Motion Previous: Projectile Motion with Air Consider a particle of mass and electric charge moving in the uniform electric and magnetic fields, and . Suppose that the fields are ``crossed'' (i.e., perpendicular to one another), so that . The force acting on the particle is given by the familiar Lorentz law: (194) where is the particle's instantaneous velocity. Hence, from Newton's second law, the particle's equation of motion can be written (195) It turns out that we can eliminate the electric field from the above equation by transforming to a different inertial frame. Thus, writing (196) Equation (195) reduces to (197) where we have made use of a standard vector identity (see Section A.10), as well as the fact that . Hence, we conclude that the addition of an electric field perpendicular to a given magnetic field simply causes the particle to drift perpendicular to both the electric and magnetic field with the fixed velocity (198) irrespective of its charge or mass. It follows that the electric field has no effect on the particle's motion in a frame of reference which is co-moving with the so-called E-cross-B velocity given above. Let us suppose that the magnetic field is directed along the -axis. As we have just seen, in the frame, the particle's equation of motion reduces to Equation (197), which can be written: (199) (200) (201) Here, (202) is the so-called cyclotron frequency. Equations (199)-(201) can be integrated to give (203) (204) (205) where we have judiciously chosen the origin of time so as to eliminate any phase offset in the arguments of the above trigonometrical functions. According to Equations (203)-(205), in the frame, our charged particle gyrates at the cyclotron frequency in the plane perpendicular to the magnetic field with some fixed speed , and drifts parallel to the magnetic field with some fixed speed . The fact that the cyclotron frequency is positive for positively charged particles, and negative for negatively charged particles, just means that oppositely charged particles gyrate in opposite directions in the plane perpendicular to the magnetic field. Equations (203)-(205) can be integrated to give (206) (207) (208) where we have judiciously chosen the origin of our coordinate system so as to eliminate any constant offsets in the above equations. Here, (209) is called the Larmor radius. Equations (206)-(208) are the equations of a spiral of radius , aligned along the direction of the magnetic field (i.e., the -direction). Figure 12: The spiral trajectory of a negatively charged particle in a magnetic field. We conclude that the general motion of a charged particle in crossed electric and magnetic field is a combination of drift [see Equation (198)] and spiral motion aligned along the direction of the magnetic field--see Figure 12. Particles drift parallel to the magnetic field with constant speeds, and gyrate at the cyclotron frequency in the plane perpendicular to the magnetic field with constant speeds. Oppositely charged particles gyrate in opposite directions. Next: Exercises Up: Multi-Dimensional Motion Previous: Projectile Motion with Air Richard Fitzpatrick 2011-03-31 The only way for a vector field to have strict spherical symmetry is for it to be purely in the radial direction. For, if it had a non-radial component then that component would have to be preserved under rotations, but you cannot construct a vector field which has that property everywhere on the surface of a sphere. I provide a proof below. (This is closely related to, but not exactly the same as, the hairy ball theorem.) So the only type of vector field which has strict spherical symmetry is a purely radial one, such as a Coulomb field. Such a field cannot be an electromagnetic wave. So it is not possible to have exactly a spherical electromagnetic wave (i.e. one with no change at all under rotations). You can have a wave which in the limit $r \rightarrow \infty$ has spherical wavefronts and is transverse, but I suppose the question is not about that limit, since it amounts to adopting a plane wave approximation for each part of the spherical wavefront. You can have an oscillating field which has spherical wavefronts, where a wavefront is a locus of a fixed value of the phase of the oscillation. Such a field is not exactly transverse everywhere. A proof of the claim (I just made up this proof; I'm adding it to see if anyone likes it or tells me it is not good enough.) Take a sphere, and put a vector $\bf E$ at some point P on it. Let's define the 'equator' of our sphere to be the great circle running through P and parallel to $\bf E$ there. Now rotate the sphere through 90 degrees, carrying P and $\bf E$ up to the north pole. The vector is pointing in a direction we shall agree to call x. Now return to the initial condition, and this time rotate the ball by 90 degrees about an axis through the poles, thus carrying P around the equator, and $\bf E$ with it. Then rotate again, carrying P up to the north pole. Now $\bf E$ is sitting at the north pole and pointing in a direction y, at right angles to the direction we got in the first rotation. But if we had been able to paint a vector field onto our sphere such that it had spherical symmetry, then these two transformations should both give no net effect on the whole sphere, and therefore both should carry $\bf E$ to a direction at the pole which would be the same in both cases. But it is not the same, so we have a contradiction, and the false step was the assumption that a vector field could be painted on the sphere in a spherically symmetric way. Working off-campus? Learn about our remote access options A strain-mediated perpendicular magnetic anisotropy (PMA) and current-induced magnetization switching via spin–orbit torque (SOT) in PbMg1/3Nb2/3O3-PbTiO3 (PMN-PT)/Ta/Pt/Co/Pt ferromagnetic heterostructures are reported. It is found that the PMA changes regularly with the preloaded lateral electric field. The SOT-based current-induced magnetization switching also shows a reversible trend via applying a lateral electric field. Domain wall propagation driven by electric field is observed directly under a fixed perpendicular magnetic field by magnetic optical Kerr (MOKE) microscope. These behaviors can be attributed to the strain from the PMN-PT substrates induced by the piezoelectric effect under electric field. Basing on the domain wall motion mechanism, repeatable resistance states of the Hall bar can be controlled by external electric field under a small auxiliary magnetic field, which enables the information recorded in the device can be programmed by voltage. This study provides a potential method to design the electric field controlled spintronic devices. The full text of this article hosted at iucr.org is unavailable due to technical difficulties. The most succinct answer is relativistic covariance. The scalar electric potential and the vector magnetic potential are the components of a four-vector - the four-potential. The electric and magnetic fields are then components of a four-tensor - the Faraday tensor. This is so that the electric and magnetic fields transform properly under a Lorentz transformation. Interestingly, when thinking in terms of spacetime and four-vectors, the electromagnetic four-force on a particle is (Minkowski) orthogonal to the particle's four-velocity. Simply put, a particle's four-velocity has constant length and thus, the four-acceleration must by (Minkowski) orthogonal, i.e., acceleration can only change the direction of the four-velocity, not the length. The Lorentz force, expressed in four-vector notation is $$\frac{dp_{\alpha}}{d\tau} = qF_{\alpha \beta}u^{\beta} $$ The left hand side is the four-force, the right hand side is the product of the charge of the particle with the contraction of the Faraday tensor and the four-velocity. For a particle at rest, the four-velocity points in the time direction and it's straight forward to show that the four-force is due to the electric field and points in a space direction, i.e., the four-force due to the electric field is orthogonal to the particle's four-velocity. Thus, we see that, in the relativistic four-vector context, both the electric and magnetic forces are orthogonal to the particle's four-velocity. To summarize, starting with the manifestly covariant four-potential and its exterior derivative, the Faraday tensor, and the Lorentz force law, we find that the 3D + 1 expression for the Lorentz force is $$\frac{d\vec p}{dt} = q(\vec E + \vec v \times \vec B)$$ $$\frac{dE}{dt} = q\vec v \cdot \vec E $$ where \$E\$ is the particle's energy. We want now to describe—mainly in a qualitative way—the motions of charges in various circumstances. Most of the interesting phenomena in which charges are moving in fields occur in very complicated situations, with many, many charges all interacting with each other. For instance, when an electromagnetic wave goes through a block of material or a plasma, billions and billions of charges are interacting with the wave and with each other. We will come to such problems later, but now we just want to discuss the much simpler problem of the motions of a single charge in a given field. We can then disregard all other charges—except, of course, those charges and currents which exist somewhere to produce the fields we will assume. We should probably ask first about the motion of a particle in a uniform electric field. At low velocities, the motion is not particularly interesting—it is just a uniform acceleration in the direction of the field. However, if the particle picks up enough energy to become relativistic, then the motion gets more complicated. But we will leave the solution for that case for you to play with. Next, we consider the motion in a uniform magnetic field with zero electric field. We have already solved this problem—one solution is that the particle goes in a circle. The magnetic force $q\FLPv\times\FLPB$ is always at right angles to the motion, so $d\FLPp/dt$ is perpendicular to $\FLPp$ and has the magnitude vp/R, where R is the radius of the circle: \begin{equation*} F=qvB=\frac{vp}{R}. \end{equation*} The radius of the circular orbit is then \begin{equation} \label{Eq:II:29:1} R=\frac{p}{qB}. \end{equation} That is only one possibility. If the particle has a component of its motion along the field direction, that motion is constant, since there can be no component of the magnetic force in the direction of the field. The general motion of a particle in a uniform magnetic field is a constant velocity parallel to $\FLPB$ and a circular motion at right angles to $\FLPB$—the trajectory is a cylindrical helix (Fig. 29–1). The radius of the helix is given by Eq. (29.1) if we replace p by $p_\perp$, the component of momentum at right angles to the field. Fig. 29–1.Motion of a particle in a uniform magnetic field. A uniform magnetic field is often used in making a "momentum analyzer," or "momentum spectrometer," for high-energy charged particles. Suppose that charged particles are shot into a uniform magnetic field at the point A in Fig. 29–2(a), the magnetic field being perpendicular to the plane of the drawing. Each particle will go into an orbit which is a circle whose radius is proportional to its momentum. If all the particles enter perpendicular to the edge of the field, they will leave the field at a distance x (from A) which is proportional to their momentum p. A counter placed at some point such as C will detect only those particles whose momentum is in an interval Δp near the momentum $p=qBx/2$. Fig. 29–2.A uniform-field momentum spectrometer with 180° focusing: (a) different momenta; (b) different angles. (The magnetic field is directed perpendicular to the plane of the figure.) It is, of course, not necessary that the particles go through 180° before they are counted, but the so-called "180° spectrometer" has a special property. It is not necessary that all the particles enter at right angles to the field edge. Figure 29–2(b) shows the trajectories of three particles, all with the same momentum but entering the field at different angles. You see that they take different trajectories, but all leave the field very close to the point C. We say that there is a "focus." Such a focusing property has the advantage that larger angles can be accepted at A—although some limit is usually imposed, as shown in the figure. A larger angular acceptance usually means that more particles are counted in a given time, decreasing the time required for a given measurement. By varying the magnetic field, or moving the counter along in x, or by using many counters to cover a range of x, the "spectrum" of momenta in the incoming beam can be measured. [By the "momentum spectrum" $f(p)$, we mean that the number of particles with momenta between p and $(p+dp)$ is $f(p)\,dp$.] Such measurements have been made, for example, to determine the distribution of energies in the β-decay of various nuclei. There are many other forms of momentum spectrometers, but we will describe just one more, which has an especially large solid angle of acceptance. It is based on the helical orbits in a uniform field, like the one shown in Fig. 29–1. Let's think of a cylindrical coordinate system—ρ,θ,z—set up with the z-axis along the direction of the field. If a particle is emitted from the origin at some angle α with respect to the z-axis, it will move along a spiral whose equation is \begin{equation*} \rho=a\sin kz,\quad\theta=bz, \end{equation*} where a, b, and k are parameters you can easily work out in terms of p, α, and the magnetic field B. If we plot the distance ρ from the axis as a function of z for a given momentum, but for several starting angles, we will get curves like the solid ones drawn in Fig. 29–3. (Remember that this is just a kind of projection of a helical trajectory.) When the angle between the axis and the starting direction is larger, the peak value of ρ is large but the longitudinal velocity is less, so the trajectories for different angles tend to come to a kind of "focus" near the point A in the figure. If we put a narrow aperture of A, particles with a range of initial angles can still get through and pass on to the axis, where they can be counted by the long detector D. Fig. 29–3.An axial-field spectrometer. Particles which leave the source at the origin with a higher momentum but at the same angles, follow the paths shown by the broken lines and do not get through the aperture at A. So the apparatus selects a small interval of momenta. The advantage over the first spectrometer described is that the aperture A—and the aperture A'—can be an annulus, so that particles which leave the source in a rather large solid angle are accepted. A large fraction of the particles from the source are used—an important advantage for weak sources or for very precise measurements. One pays a price for this advantage, however, because a large volume of uniform magnetic field is required, and this is usually only practical for low-energy particles. One way of making a uniform field, you remember, is to wind a coil on a sphere, with a surface current density proportional to the sine of the angle. You can also show that the same thing is true for an ellipsoid of rotation. So such spectrometers are often made by winding an elliptical coil on a wooden (or aluminum) frame. All that is required is that the current in each interval of axial distance Δx be the same, as shown in Fig. 29–4. Fig. 29–4.An ellipsoidal coil with equal currents in each axial interval Δx produces a uniform magnetic field inside. Particle focusing has many applications. For instance, the electrons that leave the cathode in a TV picture tube are brought to a focus at the screen—to make a fine spot. In this case, one wants to take electrons all of the same energy but with different initial angles and bring them together in a small spot. The problem is like focusing light with a lens, and devices which do the corresponding job for particles are also called lenses. Fig. 29–5.An electronic lens. The field lines shown are "lines of force," that is, of $q\FigE$. One example of an electron lens is sketched in Fig. 29–5. It is an "electrostatic" lens whose operation depends on the electric field between two adjacent electrodes. Its operation can be understood by considering what happens to a parallel beam that enters from the left. When the electrons arrive at the region a, they feel a force with a sidewise component and get a certain impulse that bends them toward the axis. You might think that they would get an equal and opposite impulse in the region b, but that is not so. By the time the electrons reach b they have gained energy and so spend less time in the region b. The forces are the same, but the time is shorter, so the impulse is less. In going through the regions a and b, there is a net axial impulse, and the electrons are bent toward a common point. In leaving the high-voltage region, the particles get another kick toward the axis. The force is outward in region c and inward in region d, but the particles stay longer in the latter region, so there is again a net impulse. For distances not too far from the axis, the total impulse through the lens is proportional to the distance from the axis (Can you see why?), and this is just the condition necessary for lens-type focusing. You can use the same arguments to show that there is focusing if the potential of the middle electrode is either positive or negative with respect to the other two. Electrostatic lenses of this type are commonly used in cathode-ray tubes and in some electron microscopes. Fig. 29–6.A magnetic lens. Fig. 29–7.Electron motion in the magnetic lens. Another kind of lens—often found in electron microscopes—is the magnetic lens sketched schematically in Fig. 29–6. A cylindrically symmetric electromagnet has very sharp circular pole tips which produce a strong, nonuniform field in a small region. Electrons which travel vertically through this region are focused. You can understand the mechanism by looking at the magnified view of the pole-tip region drawn in Fig. 29–7. Consider two electrons a and b that leave the source S at some angle with respect to the axis. As electron a reaches the beginning of the field, it is deflected away from you by the horizontal component of the field. But then it will have a lateral velocity, so that when it passes through the strong vertical field, it will get an impulse toward the axis. Its lateral motion is taken out by the magnetic force as it leaves the field, so the net effect is an impulse toward the axis, plus a "rotation" about the axis. All the forces on particle b are opposite, so it also is deflected toward the axis. In the figure, the divergent electrons are brought into parallel paths. The action is like a lens with an object at the focal point. Another similar lens upstream can be used to focus the electrons back to a single point, making an image of the source S. Fig. 29–8.The resolution of a microscope is limited by the angle subtended from the source. You know that electron microscopes can "see" objects too small to be seen by optical microscopes. We discussed in Chapter 30 of Vol. I the basic limitations of any optical system due to diffraction of the lens opening. If a lens opening subtends the angle 2θ from a source (see Fig. 29–8), two neighboring spots at the source cannot be seen as separate if they are closer than about \begin{equation*} \delta\approx\frac{\lambda}{\sin\theta}, \end{equation*} where λ is the wavelength of the light. With the best optical microscope, θ approaches the theoretical limit of 90°, so δ is about equal to λ, or approximately 5000 angstroms. The same limitation would also apply to an electron microscope, but there the wavelength is—for 50-kilovolt electrons—about 0.05 angstrom. If one could use a lens opening of near 30°, it would be possible to see objects only $\tfrac{1}{5}$ of an angstrom apart. Since the atoms in molecules are typically 1 or 2 angstroms apart, we could get photographs of molecules. Biology would be easy; we would have a photograph of the DNA structure. What a tremendous thing that would be! Most of present-day research in molecular biology is an attempt to figure out the shapes of complex organic molecules. If we could only see them! Unfortunately, the best resolving power that has been achieved in an electron microscope is more like 20 angstroms. The reason is that no one has yet designed a lens with a large opening. All lenses have "spherical aberration," which means that rays at large angles from the axis have a different point of focus than the rays nearer the axis, as shown in Fig. 29–9. By special techniques, optical microscope lenses can be made with a negligible spherical aberration, but no one has yet been able to make an electron lens which avoids spherical aberration. Fig. 29–9.Spherical aberration of a lens. In fact, one can show that any electrostatic or magnetic lens of the types we have described must have an irreducible amount of spherical aberration. This aberration—together with diffraction—limits the resolving power of electron microscopes to their present value. The limitation we have mentioned does not apply to electric and magnetic fields which are not axially symmetric or which are not constant in time. Perhaps some day someone will think of a new kind of electron lens that will overcome the inherent aberration of the simple electron lens. Then we will be able to photograph atoms directly. Perhaps one day chemical compounds will be analyzed by looking at the positions of the atoms rather than by looking at the color of some precipitate! Magnetic fields are also used to produce special particle trajectories in high energy particle accelerators. Machines like the cyclotron and synchrotron bring particles to high energies by passing the particles repeatedly through a strong electric field. The particles are held in their cyclic orbits by a magnetic field. We have seen that a particle in a uniform magnetic field will go in a circular orbit. This, however, is true only for a perfectly uniform field. Imagine a field B which is nearly uniform over a large area but which is slightly stronger in one region than in another. If we put a particle of momentum p in this field, it will go in a nearly circular orbit with the radius $R=p/qB$. The radius of curvature will, however, be slightly smaller in the region where the field is stronger. The orbit is not a closed circle but will "walk" through the field, as shown in Fig. 29–10. We can, if we wish, consider that the slight "error" in the field produces an extra angular kick which sends the particle off on a new track. If the particles are to make millions of revolutions in an accelerator, some kind of "radial focusing" is needed which will tend to keep the trajectories close to some design orbit. Fig. 29–10.Particle motion in a slightly nonuniform field. Another difficulty with a uniform field is that the particles do not remain in a plane. If they start out with the slightest angle—or are given a slight angle by any small error in the field—they will go in a helical path that will eventually take them into the magnet pole or the ceiling or floor of the vacuum tank. Some arrangement must be made to inhibit such vertical drifts; the field must provide "vertical focusing" as well as radial focusing. Fig. 29–11.Radial motion of a particle in a magnetic field with a large positive slope. One would, at first, guess that radial focusing could be provided by making a magnetic field which increases with increasing distance from the center of the design path. Then if a particle goes out to a large radius, it will be in a stronger field which will bend it back toward the correct radius. If it goes to too small a radius, the bending will be less, and it will be returned toward the design radius. If a particle is once started at some angle with respect to the ideal circle, it will oscillate about the ideal circular orbit, as shown in Fig. 29–11. The radial focusing would keep the particles near the circular path. Fig. 29–12.Radial motion of a particle in a magnetic field with a small negative slope. Actually there is still some radial focusing even with the opposite field slope. This can happen if the radius of curvature of the trajectory does not increase more rapidly than the increase in the distance of the particle from the center of the field. The particle orbits will be as drawn in Fig. 29–12. If the gradient of the field is too large, however, the orbits will not return to the design radius but will spiral inward or outward, as shown in Fig. 29–13. Fig. 29–13.Radial motion of a particle in a magnetic field with a large negative slope. We usually describe the slope of the field in terms of the "relative gradient" or field index, n: \begin{equation} \label{Eq:II:29:2} n=\frac{dB/B}{dr/r}. \end{equation} A guide field gives radial focusing if this relative gradient is greater than -1. Fig. 29–14.A vertical guide field as seen in a cross section perpendicular to the orbits. A radial field gradient will also produce vertical forces on the particles. Suppose we have a field that is stronger nearer to the center of the orbit and weaker at the outside. A vertical cross section of the magnet at right angles to the orbit might be as shown in Fig. 29–14. (For protons the orbits would be coming out of the page.) If the field is to be stronger to the left and weaker to the right, the lines of the magnetic field must be curved as shown. We can see that this must be so by using the law that the circulation of $\FLPB$ is zero in free space. If we take coordinates as shown in the figure, then \begin{equation} (\FLPcurl{\FLPB})_y=\ddp{B_x}{z}-\ddp{B_z}{x}=0,otag \end{equation} or \begin{equation} \label{Eq:II:29:3} \ddp{B_x}{z}=\ddp{B_z}{x}. \end{equation} Since we assume that $\ddpl{B_z}{x}$ is negative, there must be an equal negative $\ddpl{B_x}{z}$. If the "nominal" plane of the orbit is a plane of symmetry where $B_x=0$, then the radial component B_x will be negative above the plane and positive below. The lines must be curved as shown. Such a field will have vertical focusing properties. Imagine a proton that is travelling more or less parallel to the central orbit but above it. The horizontal component of $\FLPB$ will exert a downward force on it. If the proton is below the central orbit, the force is reversed. So there is an effective "restoring force" toward the central orbit. From our arguments there will be vertical focusing, provided that the vertical field decreases with increasing radius; but if the field gradient is positive, there will be "vertical defocusing." So for vertical focusing, the field index n must be less than zero. We found above that for radial focusing n had to be greater than -1. The two conditions together give the condition that \begin{equation*} -1